Probabilistic Methods in Combinatorics

Instructor: Oliver Janzer

Assignment 4

To solve for the Example class on 18th March. Submit the solution of Problem 1 by Sunday 16th March if you wish feedback on it.

The solution of each problem should be no longer than one page!

Problem 1. Let $v_1, \ldots, v_n \in \mathbb{R}^n$ be vectors with Euclidean norm 1. Show that there exist $\epsilon_1, \ldots, \epsilon_n \in \{-1, +1\}$ such that

$$||\epsilon_1 v_1 + \ldots + \epsilon_n v_n|| \leq \sqrt{n},$$

where $||.||$ is the Euclidean norm.

Problem 2. Let H be a bipartite graph with parts A and B such that all vertices in B have degree at most r. Show that there exists a positive constant $c = c(H)$ depending only on H such that any graph G on n vertices with at least $cn^{2-1/r}$ edges contains H as a subgraph.